November 2017 subject reports

Mathematical Studies SL

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0–14	15–27	28–41	42–54	55–67	68–79	80–100

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–4	5–6	7–8	9–11	12–14	15–16	17–20

The range and suitability of the work submitted

As in previous sessions the majority of candidates opted for a statistical analysis project. There were a few projects that should have been actively discouraged by the teacher as they lacked any originality. There were also some projects that seemed to be more like a homework assignment than a Mathematical Studies SL project. These projects did not show the time requirement for an MSSL project and also showed that teachers did not give sufficient guidance to their candidates. There was a wide variety in the quality levels across schools. Some schools and teachers seemed to understand the criteria and the expectations for the project quite well, whereas, in other centres, projects were generally weak, data collection was sparse and the teacher did not seem to understand the assessment criteria. Most of the candidates used surveys or internet referenced sources to collect their data. It was pleasing to frequently see sources referenced and bibliographies given.

Projects had either the raw data at the beginning or in an appendix and it was possible to verify calculations. The vast majority of projects had structure and developed logically. Most projects had at least some appropriate notation and terminology. More candidates this session attempted to define their variables. The conclusions drawn were generally consistent with the results. Validity was, as always, the criterion least well addressed although there were great improvements in the attempts to discuss limitations. Unfortunately, there were still some careless errors in calculations, notation and terminology. Teachers are also encouraged to make comments throughout the project in the margins and check the accuracy of the mathematics.

Candidate performance against each criterion

A: Candidates, generally, were able to achieve level 2 in this criterion, as projects contained an aim, a title and a plan, although at times a little brief. Candidates usually mentioned some the mathematical processes they would use. At times, there are processes not mentioned in the plan that are carried out in the analysis and vice versa. This precludes the candidates achieving more than level 2. Giving reasons for the mathematical processes appears to be challenging for the candidates but they have to include this in order to be awarded full marks.

B: Some candidates did not show the raw data collected so it was not possible to verify all of the tables or calculations. In general, candidates understand this objective well. Candidates are able to gather raw data either by personal collection or from the internet and organize it in a manner appropriate for analysis. Very few candidates seem aware of how to collect a random sample. Most samples are convenience samples or the candidate believes that if they wander down a hallway and ask whoever passes by, that this is a random sample, and unfortunately many teachers seem to think this as well. Most candidates are able to earn level 2 for this criterion but not level 3. Frequently, the data is just too sparse for the intended analysis, especially if the χ^2 test is an intended process. The data is also, too frequently, very simple in nature.

C: Most candidates and teachers were aware of the need to present some sample calculations by hand, or to present their calculations in the context of the formula. However, many teachers and candidates did not seem to focus on the requirement for relevance to earn level 3 in this criterion. More candidates than previously interpreted the simple processes, bar charts and pie charts, quite well. The stronger candidates who carried out a variety of mathematical processes did not always achieve level 5 because either the line of best fit was found algebraically then not used or too many expected values in the chi squared test were less than 5. Candidates using a 2 by 2 matrix with degree of freedom 1, in most cases, recognized the need to apply Yates' continuity correction. Many candidates used Excel and some added a trend line, a calculation of the regression equation and a calculation of r^2 without explanation or justification. These poor practices preclude higher marks. Too few candidates proceeded with regression in the logical order of scatter plot, calculation of the coefficient, followed by a regression line if appropriate. In addition, if the regression line is not used in any meaningful way, it is hard to understand the purpose of the calculation, whether it is mathematically valid or not. Teachers should be aware that repeating the same process several times does not count as multiple processes.

D: Most candidates drew at least one interpretation consistent with their results. Candidates commonly earned level 2 in this criterion. In the better projects, candidates presented partial conclusions as they went along, and then summarized these at the end. Few candidates earned level 3 for this criterion because the projects were too simple. Candidates should be discouraged from making unsubstantiated conjectures about the reasons for their findings. Many teachers awarded 2 marks for a single consistent conclusion.

E: It was usually the stronger candidates who commented meaningfully upon the processes used and the results found. Some went on to discuss the limitations of their results. Many candidates commented on the validity of their data in a manner that went beyond “I needed more data.” Some candidates think their processes are valid if they have checked their calculations or they have performed their analysis on Excel. It was common for valid and accurate to be treated as synonyms. Some candidates made no attempt to fulfil this criterion.

F: Most of the projects had some structure and were developed logically. A few projects lacked explanation at each stage. Others had graphs and mathematical processes out of order. Bibliographies/referenced sources were often seen in an appendix. Level 3 was not achieved mainly because, although the project was quite good, it was too simple. For those performing a χ^2 test, level 3 was often not achieved if there was a lack of explanation on how the categories were subdivided. Teachers and candidates seem unaware of the need to clearly explain how their data was divided up for the χ^2 test.

G: Most candidates were able to earn one of the two marks for this criterion but few candidates earned both marks. Candidates should be taught how to use a simple equation editor and the importance of defining their variables.

Recommendations for the teaching of future candidates

Teachers should:

- ensure all mathematical processes used in the project are described in the introduction,
- make sure that the candidates present all raw data collected in the body of the project or in the appendix,
- ensure the simple processes used are meaningful and relevant to the task,
- ensure that the candidates define the variables,
- ensure that candidates show all calculations that lead up to the result,
- ensure, when found, that the equation of the regression line is used,
- explain sampling to the candidates,
- encourage candidates to show calculations by hand even if they are making use of technology such as Excel,
- explain how the categories for the χ^2 test were subdivided,
- help the candidates to understand how to address validity,
- show the candidates how to use equation editor such as the one in MS Word or MathType,
- make sure that all candidates read the assessment criteria and are fully aware of what they demand,
- explicitly provide evidence, with the IA projects (preferably by annotating the pages directly) for awarding the different levels of achievement for the criteria,
- give candidates examples that show good work, mediocre work and bad work, so they can better understand the difference between them,
- monitor candidate work, and give candidates suggestions about how to increase the sophistication of their analysis,
- teachers should select the level in each criterion that matches their comments,
- teachers should indicate which mathematical processes have been checked.

Further comments

It would greatly help the moderation process if teachers wrote comments related to each criterion specifically against the evidence in the body of the projects; some schools did and this was extremely helpful for the moderator to understand, and where possible confirm, the teacher's application of the criteria.

Teachers could, as an alternative, write their comments in the text box at the point of upload, stating the page where the evidence for awarding each level for the different criteria is located.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–11	12–22	23–34	35–46	47–57	58–69	70–90

The areas of the programme and examination which appeared difficult for the candidates

Prior Learning

A surprising number of candidates were unable to recognize an application of the Pythagorean Theorem.

Topic 1: Number and algebra

Many candidates were unable to convert between measurement in seconds to minutes and misunderstood the quantity of a million. Candidates should use their graphic display calculator (GDC) to avoid carless errors in manually manipulating the algebra. Many candidates ignored or were unable to follow the instruction to give answers to two decimal places. Although not in the syllabus there were many candidates using “simple interest” instead of “compound interest” or unable to correctly substitute into the formula.

Topic 2: Descriptive Statistics

Candidates confused the different measures of central tendency and many were unable to find the mean from a frequency table.

Topic 3: Logic sets and probability

Candidates had difficulty filling in the Venn diagram and using their values to find conditional probability and determining if the events were independent.

Topic 4: Statistical application

Many candidates were unable draw the regression line from the equation and few recognized that extrapolation is unreliable.

Topic 6: Mathematical Models

Only the very best candidates were able to find the correct values of a and b in a quadratic function and likewise the parameters of the exponential function.

Topic 7: Introductory Differential Calculus

Only the very best candidates correctly differentiated and found the point that had a given gradient.

The areas of the programme and examination in which candidates appeared well prepared

Prior Learning

Almost all candidates could find the midpoint.

Topic 2: Descriptive Statistics

Almost all candidates were able to draw correctly the whiskers on a box-and-whisker diagram.

Topic 3: Logic sets and probability

Candidates were able to fill in the truth table and rewrite in words a compound statement given in symbolic form.

Topic 5: Geometry and trigonometry

Almost all candidates could find the gradient of a linear function. Many were able to recognize and correctly use the law of sines.

Topic 7: Introductory Differential Calculus

Almost all candidates could find the derivative of a cubic function.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Box-and-whisker plots

This question was attempted by almost every candidate. Many candidates plotted their mean instead of the median. Frequently a mark was lost for not using a ruler; as per the command term, “draw”, a ruler is required for accurate drawing.

Question 2: Midpoint and gradient of a linear function

There were many correct answers for the midpoint and the gradient. Most candidates found the negative reciprocal for the perpendicular gradient but fewer could find the equation of the line. Many candidates substituted points into $y = mx + c$ in order to find c. Unfortunately, often they did not substitute the correct point. Many did not realize that they only had to use their answer from part (a).

Question 3: SI units and measurements

Perhaps the large values of the distances in space were unfamiliar for candidates. There were not many fully correct answers for this question. Some candidates initially divided 300000 by 60 to begin with and this lead to the wrong answer. Others did not use the correct numerator and some divided the wrong way around. There were more correct answers for part (b). Even here, the word “million” seemed to confuse some candidates and their answer was written as 3.06×10^{15}.

Question 4: Logic

Most candidates included “if …. then” in their answer. Some wrote “and” instead of “or”. The truth table was not so well attempted and quite a few candidates knew that it was the contrapositive.

Question 5: Scatter diagrams and regression lines

Not all candidates realized that they had to use the equation given in the question and so some wasted time by finding all the points on the graph in order to find the mean. Not many candidates could draw the line of best fit. Some managed to put it through the mean point (or thereabouts) but few had their line going through $(0, 15)$. Many put the line through the origin. A lot of candidates thought that the correlation was positive but a few thought it was negative or there was no correlation. Some candidates knew about extrapolation but many wrote down a non-mathematical reason here or thought it was an outlier.

Question 6: Pythagorean Theorem, ratio and solving simultaneous equations

Surprisingly few candidates used the Pythagorean theorem to find the diagonal of a rectangle. It was more common to see the ratio of horizontal to vertical reversed. Very few candidates gave a correct answer for the dimensions of the computer screen. Most did not even attempt it.

Question 7: Venn diagrams

The majority of candidates wrote wrong values in the Venn diagram. Some managed to gain follow through marks for the probability. Correct answers explaining why the events were independent were rare. Most candidates just wrote "they are independent" or "they are not independent" without giving any reason; or they gave a wrong reason or one based on the context, lacking mathematical reasoning.

Question 8: Currency exchange

This was one of the better attempted questions in the paper although many candidates lost marks due to incorrect rounding as the direction in the question, to round to two decimal places, was ignored. Only the best were able to attempt the last part, with some candidates dividing the currency exchange the wrong way around.

Question 9: Compound interest

Some candidates managed to find the correct original price given the sale price. Few candidates managed to get the compound interest amount correct. If they used the formula then many put 6 (the number of months of the loan) instead of 12 for "k" (the number of times per year the loan is compounded). If they used their GDC, they often put the wrong number in P/Y or N.

Question 10: Trigonometry

Many candidates did not attempt this question. Some used right-angled trigonometry instead of the sine rule. Most of the candidates who found an answer also included the units. Writing down the angle of elevation was sometimes the only part of this question that was attempted. There were many wrong answers given.

Question 11: Quadratic function

Some candidates did manage to find the equation of the axis of symmetry correctly. Quite a few more found the correct y-intercept. Very few tried to find the values for a and b.

Question 12: Exponential model

Interpreting the context of the horizontal asymptote led to the most common answer: “the maximum temperature” followed by a correct interpretation then an indication that this was the initial temperature. In the second part, many candidates made incorrect substitutions into the exponential equation given the LHS was $T(t)$. There were not many correct answers for the last part of this question.

Question 13: Normal distribution

This was either done very well with clear understanding and occasionally incorrect calculator input, or answered poorly. Often there was no response.

Question 14: Polynomial differentiation and finding the coordinates given the gradient

A lot of candidates managed to find the derivative correctly. Others managed to differentiate only the cubic term correctly. Some knew to equate their derivative equal to the given gradient but then could not solve the equation. Many of those who did solve the equation only gave $x = 1$ as their final answer rather than $(1, 4)$.

Question 15: Mathematical models

Many candidates could complete the first two parts of this question although 552 and 662 were also seen. Fewer managed to write the correct expression for part (c) and correct answers for the last part were few and far between.

Recommendations and guidance for the teaching of future candidates

- Candidates should be familiar with command terms, for example that "draw" means to use a ruler if the diagram is to be straight at any point.
- Follow the specific instructions on the question when specified e.g. “give a reason why…” or “give answers correct to two decimal places”.
- Use the functions of the GDC more frequently, for example to solve simultaneous equations.
- Practise and review past papers in order to become familiar with the structure and content of the assessment.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–12	13–25	26–40	41–51	52–62	63–73	74–90

General comments

Overall the paper was accessible with many candidates accessing marks in the higher end of the spectrum. For the most part, candidates were not required to use sophisticated mathematical processes, nor work with complicated functions. The difficulty in this paper lay in its breadth, asking candidates to process the given information and then apply mathematics in unfamiliar settings.

The areas of the programme and examination which appeared difficult for the candidates

- Reading and interpreting data collated in a table.
- Understanding what is meant by the command term ‘Show that’.
- Using the given information in a question.
- Compound probability events.
- Conditional probability.
- Representing a number in exact form.
- Finding the zeros of a function.
- Increasing functions.
- Labelling axes of graphs and sketching accurate smooth curves.
- Using and interpreting a mathematical model in a contextualized situation.

The areas of the programme and examination in which candidates appeared well prepared

- χ^2 test for independence.
- Arithmetic sequence and series.
- Application of the Pythagorean Theorem.
- Cosine rule.
- Area of a triangle.
- Calculating percentage error.
- Calculating simple probability.
- Algebraic manipulation.
- Finding the derivative of a function.
- Use of GDC to find the coordinates of key features of a function.
- Calculating the volume of a cone.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

A surprising number of candidates were not able to identify the data as discrete. A wide variety of answers were seen for the mean and standard deviation of a grouped frequency distribution. Candidates calculated the mean of frequencies, the mean of frequencies in the modal class and the mean of the class midpoints. No doubt some candidates had difficulty interpreting data in the table, while others did not include the class frequencies in their calculation. Although the mean and standard deviation are expected to be calculated using the GDC, some candidates were able to gain at least one mark for attempting to use the formula for the mean. Candidates appeared to understand the χ^2 test for independence. The null hypothesis was stated correctly, expected value and degrees of freedom found. Most found the χ^2 statistic and associated *p*-value, and followed through with a correct interpretation. Only the weaker candidates compared the level of significance with the critical value. Some lost a mark in part (f)(i) as they simply wrote down their *p*-value as 0.06. Candidates should be advised to write down the full value seen in the graphic display calculator. Answers given to one significant figure are considered incorrect and accrue no marks, as per the instructions on the front cover that answers should be exact or correct to three significant figures.

Question 2

Candidates were able to find the distance ran in the third training session. In part (a)(ii), it was common to see the correctly substituted expression for the n^{th} training session, but further working developed revealed that candidates did not fully understand what was being asked. It was pleasing to see most candidates set up an equation or inequality to find the value of k. Candidates employing a “trial and error” method were also successful in this part. Answers were usually given in the form of an integer. The formula for the arithmetic series was correctly substituted in part (c), with a correct answer in metres seen. Not all candidates expressed their answer in kilometres. This was despite ‘kilometres’ written in bold in the question. Future candidates should take note, that words in bold emphasize an important instruction or piece of information. Stronger candidates appeared to have little difficulty with the mix of units throughout this question. In most cases the units were either correct or omitted. Some did not recognize the geometric relationship in parts (d) and (e), treating the distance of Carlos as an arithmetic progression. Other common errors included use of the compound interest formula or incorrect value of the common ratio, *r*. Those candidates who used a list in these questions were less successful. This was sometimes due to a clerical error. Candidates are advised to use appropriate formulae for sequences and series. Manual calculations are all too often prone to transcription errors and inaccuracy. Use of $r = 20$ produced an unrealistic answer, while $r = 0.2$ did not satisfy the demand of the question, that the distance *‘increased’*. Candidates should be encouraged to question unrealistic values and review their results.

Question 3

Overall this question was well answered. Correct use of the Pythagorean Theorem resulted in the given distance. A number of candidates lost the final mark for not showing both the unrounded and rounded answers in this part. When candidates are required to reach a given answer to a specified accuracy, they must first show the value they obtain correct to a higher degree of accuracy and then write down the given value so that these can be seen to be the same. Those who used the cosine rule to find $\mathrm{B\hat{C}D}$ were usually successful in this task. Use of radians did not appear to be an issue. Few candidates lost marks for assuming ABC was a right-angled triangle. Candidates were conscious to include units, although this was not always done consistently in all parts. Calculation of the area of the triangles was carried out successfully, although some candidates did not use the easier formula for a right-angle triangle. Most calculated the percentage error correctly. The omission of the absolute value sign did not incur a penalty, but a negative answer lost the final mark. Relative to past sessions, candidates appeared to be well prepared for percentage error problems.

Question 4

Candidates were able to find the elementary probability in part (a). Those who simplified their fraction to $\frac{17}{30}$, often made an error with 'non-replacement' by incorrectly calculating $\frac{17}{30}\times\frac{16}{29}$ in part (b). It is recommended that candidates write probabilities as un-simplified fractions. In order to minimize the possibility of error, future candidates should be guided to write probabilities as un-simplified fractions. Candidates had no difficulty with the complimentary probabilities in the tree diagram. Stronger candidates were able to correctly place the probability for an adult who is allergic to nuts. Candidates were able to follow through from their diagram to find the probability a candidate is allergic to nuts and the liquid turned blue. The compound probability event in part (e) proved more difficult. Consistent with past sessions, conditional probability in part (f) was one of the discriminators in this exam. Finding the correct numerator for the conditional event proved to be quite elusive. Candidates who wrote down and used the conditional probability formula were more successful in this part. Estimating the number of employees who were allergic to nuts was challenging, with many not realizing the relationship between parts (f) and (g). Candidates should be encouraged to interpret the context of the question and not just carry out an algorithm.

Question 5

It was clear that many candidates were not sure how to find the zeros of the function. A common mistake was to evaluate the function at $x=0$. There was also an over dependence on technology to find the zeros of the function. This precluded them from finding the exact value of the zeros. Those who did use their GDC, did so to good effect by using the 'zero' function rather than the crudeness of the 'trace' feature. It was disappointing that candidates did not understand nor take note of the command to express the zero in exact form. Most candidates were able to expand $f(x)$, but arithmetic errors were quite common. The attempts at expanding an algebraic expression demonstrate that teachers should be attentive to the Prior Learning topics in the Mathematical Studies SL Guide. Examination questions will assume knowledge of these topics. Candidates were able to differentiate their $f(x)$. Only the strongest candidates obtained full marks for using their equation to find the correct interval for which the curve is increasing. Some found correct endpoints but did not use correct mathematical notation to define the interval. This part proved to be a discriminator and was a testing concept for the majority. As is previous examination sessions, many did not label their axes in part (d). Candidates were conscious of drawing a smooth curve within the given domain. Although not always successful in this task, the intercepts and local maximum and minimum points were within reasonable tolerance. Although a few appeared to understand the demands of 'draw', candidates should be made aware of the distinctions between sketch and draw. It was pleasing to see candidates successfully use their GDC to find the coordinates of the point of intersection of the two curves.

Question 6

Although many correctly substituted into the volume of a cone formula, the final mark was not always accessible. Once again, the command 'Show that' proved troubling for candidates with many not stating both unrounded and rounded values. Finding the radius of the hemisphere in part (b) was either all or nothing. Candidates who equated the volume of the hemisphere to $225\ \text{cm}^3$ were usually successful in this task. The remainder of this question proved quite challenging, with many struggling to apply their knowledge to a non-routine question. In part (d), most candidates were not able to show that there was $20\ \text{cm}^3$ of orange paste. Many used the known volume to find the radius of the orange paste. This value of the radius was in turn used to show that the volume is $20\ \text{cm}^3$; circular logic is a flawed approach and earned no marks. Although candidates were often able to find the cost of the orange paste, many were not able to find the cost of the chocolate mousse. In part (f) candidates were more successful in finding the equation for the number of desserts. Those who were able to find two correct equations were usually successful in finding the number of regular desserts.

Recommendations and guidance for the teaching of future candidates

- Although the candidates appeared to take care in the accuracy of their calculations and answers, this is a point that requires constant reminder. Premature rounding can be an issue for multi part questions. Candidates should show and use unrounded answers as much as possible.
- If units are included in the context of the question they should be including in answers as necessary.
- Read each question carefully. Take note of: the information to be used; the specified method; any level of accuracy required for the final answer.
- Understand the commands such as "Find", "Show that", "Sketch", "Draw", "Calculate" etc.
- Encourage candidates to show all calculations and display the steps they make.
- The 'Show that' command requires candidates to state both the unrounded and final answers. Substituting in the known value is 'reverse engineering' and invalidates the process.
- Candidates need to be competent in using mathematical techniques.
- Candidates should know how to utilize the advanced features of the GDC. This includes solving a polynomial, solving simultaneous equations and performing statistical tests. Candidates should know how to find the measures of central tendency and dispersion for both simple and grouped data. Care must be taken when choosing an appropriate graphing window. Failure to do so may result in the candidate not

identifying important features of the function.

- Teachers should help candidates understand the default settings in their calculator.
- Candidates should be conversant with appropriate terminology for each area of the course. For example 'range' in functions has a different meaning to that in statistics.
- Candidates are to be encouraged to understand the concepts behind the algorithms. This is particularly important in differential calculus.
- Where possible use diagrams and sketches to illustrate the information.
- Start each question on a new page.
- Give consideration to the weight of a question. Lengthy explanations are not necessary when the question is only worth one or two marks.
- A graph should be drawn/sketched accurately, axes labelled and scaled. Important features in a sketch should reflect their correct location.
- Be careful to note the information that is given in the question. Full follow through marks may not be awarded for failing to use this information.
- Develop a solid understanding of the properties of geometric shapes.
- Ensure candidates are familiar with the Prior Learning topics as documented in the course guide.
- Read questions carefully, taking special note of emboldened words.
- Carefully reflect on the reasonableness of an answer.
- Be familiar with the information given in the Mathematical studies SL formula booklet.